Near Outage Limit Space-Time Coding for MIMO channels

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***Abstract*— We study simple space-time coding techniques for multiple-input multiple-output (MIMO) quasi-static channels (2Tx and 4Tx) capable of achieving near outage limit performance. The core of our space-time code is an h-π-diagonal state multiplexer that guarantees full diversity and a quasi-optimal coding gain on the MIMO channel. The whole range of word error probability is attained at signal-to-noise ratios extremely close to theoretical limits. In addition, at a fixed signal-to-noise ratio, the word error probability is insensitive to the block length.**

I. Introduction

The MIMO technology is currently a great success in recent wireless communications systems, mainly in some standards and custom-enhanced versions of IEEE 802.11a/b/g. The most famous space-time block coding (STBC) technique is the scheme proposed by Alamouti [1]. It can be easily shown that bit error probability conditioned on the channel realization for Alamouti STBC on a 2×1 MIMO channel is $P_{eb} = Q(\sqrt{y\gamma}) = f(y)$, where γ is the signal-to-noise ratio per bit, y is a second order Nakagami distributed random variable [20] representing the fading coefficients after orthogonal combining at the receiver, and $Q(x)$ is the Gaussian tail function [20]. Consider a frame of length n bits transmitted on a quasi-static MIMO channel using Alamouti scheme. Then, the frame error rate P_e is

$$P_e = \int_0^{+\infty} \left[1 - (1 - f(y))^n\right] p(y) dy$$

By upper-bounding $1-(1-x)^n$ with $\min(1, nx)$ in the interval $[0, 1]$, and after cutting the integral into two parts at the point $y = a$ defined by $nf(a) = 1$ with $\gamma >> 1$, it can be shown that

$$P_e \leq \frac{\left(2\log^2(\frac{n}{2}) + 4\log(\frac{n}{2}) + 4\right)}{\gamma^2}$$

Similarly, P_e can be lower-bounded by a quantity that varies as $\log^2(n)$. In general, for any *uncoded* STBC, the frame error probability will increase as $\log^d(n)$ where d is the diversity order. In order to approach the outage probability limit [19][5], the frame error rate of any given coding scheme should be independent of the block length [14], [15]. Therefore, such space-time coding techniques will fail in approaching the outage capacity limit of the quasi-static MIMO channel. Algebraic space-time codes [10] and any convolutionally/algebraically coded STBC also fail in approaching the outage limit. Hence, our objectives are

- Design a space-time code based on state multiplexing [8] and turbo encoding [4][3] in order to achieve near outage limit performance.
- Control the detection/decoding complexity and propose relatively low complexity schemes.
- Make the word error probability insensitive to the block length. This is the interleaving gain of turbo codes translated to the field of non-ergodic fading channels as discovered in [14], [15].

This work is a direct application of the work in [8] to MIMO channels. Closely related research can be found in [2][9][16] and other non-cited references due to the lack of space.

II. Encoder and Channel Structures

The physical channel considered in this paper is a quasi-static frequency non-selective MIMO channel with n_t transmit antennas and n_r receive antennas. The channel state information is only assumed at the receiver side for coherent detection. The MIMO channel model is

$$y = zSH + \xi \qquad (1)$$

where $z \in \mathbb{Z}[i]^{N_t}$ is a QAM modulation symbol vector, S is a linear precoding $N_t \times N_t$ unitary matrix (simply called *rotation*), H is the $N_t \times N_r$ channel matrix modeling fading coefficients between transmit and receive antennas. The additive white Gaussian noise ξ is assumed to be circularly symmetric with zero mean. The dimension of transmitted vectors z is $N_t = sn_t$, where s is the time spreading factor of the rotation S. Similarly, we have $N_r = sn_r$.

Digital transmission is made as follows: Uniformly distributed information bits are fed to a binary parallel turbo encoder. Coded bits $\{c_i\}$ are then Gray mapped into QAM symbols and transmitted on the MIMO channel given by (1). The coherent MIMO detector computes an extrinsic information $\Xi(c_i)$ based on the knowledge of H, the received vector y, and independent *a priori* information $\alpha(c_j)$ for all coded bits. Without loss of generality, we restrict our study to turbo codes with two identical recursive systematic convolutional (RSC) constituents separated by a pseudo-random interleaver π of size N. The turbo coding rate is $R_c \in (0, 1)$. The transmitted information rate is equal to $R = R_c n_t \log_2 M$ bits per channel use, where M is the cardinality of the bidimensional QAM constellation. In the sequel, we will call *BO-channel* the binary-oriented channel with input c_i and output $\Xi(c_i)$ as observed by the turbo encoder and the turbo decoder. Pseudo-random interleaving is performed prior to QAM mapping in order to enable iterative probabilistic MIMO detection [7][6] of the BO-channel. A total of n_t independent pseudo-random binary interleavers are applied and considered to be an intrinsic part of the BO-channel.

Definition 1: Under the genie condition (i.e. perfect *a priori* information) in the BO-channel, the number of independent binary-input non-ergodic fading sub-channels is denoted by D_{st} and called the *state diversity*.

Let $\omega_H(c)$ denote the Hamming weight of a turbo codeword c. We write $\omega_H(c) = \sum_{i=1}^{D_{st}} \omega_i$, where ω_i is the partial Hamming weight transmitted on the binary-input sub-channel i within the BO-channel. The state diversity $d_{st}(c)$ achieved by the codeword c is the number of non-zero partial weights. For a given transmitter structure, the achievable state diversity is $d_{st} = \min_{c\neq 0} d_{st}(c)$. State diversity is upper-bounded by [18][17]

$$d_{st} \leq \lfloor D_{st}(1-R_c)+1 \rfloor \leq D_{st} \tag{2}$$

Definition 2: For a quasi-static MIMO channel, the *channel diversity* is defined as $D_{ch} = n_t n_r$, which is equal to the intrinsic diversity order of the physical channel.

For a given transmitter structure, the achievable channel diversity is $d_{ch} = \lim_{SNR\rightarrow+\infty} -\log(P_e)/\log(SNR)$, where SNR is the signal-to-noise ratio and P_e is the error probability. Channel diversity is upper-bounded by [11][12]

$$d_{ch} \leq \min\left(s n_r \left\lfloor \frac{n_t}{s}(1-R_c)+1 \right\rfloor,\ D_{ch} \right) \tag{3}$$

With a judicious choice of an error-correcting code and a linear precoder, maximum diversity is easily attained ($d_{ch} = D_{ch}$). In general, a Nakagami distribution of order D_{ch}/D_{st} is associated to each binary-input sub-channel embedded within the BO-channel. To illustrate the above definitions, we list the following examples:

- For $n_t = 2$, $n_r = 1$, $D_{ch} = 2$, and without rotation ($s = 1$). We get $D_{st} = 2$.
- For $n_t = 2$, $n_r = 2$, $D_{ch} = 4$. Without rotation ($s = 1$), we have $D_{st} = 2$. With a cyclotomic rotation ($s = 2$), we get $D_{st} = 1$.
- For $n_t = 4$, $n_r = 2$, $D_{ch} = 8$. Without rotation ($s = 1$), we have $D_{st} = 4$. With a cyclotomic DNA rotation ($s = 2$), we get $D_{st} = 2$.

The capacity-versus-outage approach is considered in this paper. For a given signal-to-noise ratio, the outage limit in terms of word error probability is given by $P(I < R)$, where I is the instantaneous mutual information between z and y per channel use, i.e. $I = I(H) = \frac{I(z;y|H)}{s}$ bits.

III. Code Multiplexing over Channel States

In this paper, we restrict our study to the case of double state diversity $D_{st} = 2$ and half-rate turbo code $R_c = 1/2$. It should be generalized without much difficulty to other values of R_c and D_{st}. The systematic output of the turbo code is denoted by s_1 and its Hamming weight by ω. The parity bit generated by the first (resp. the second) RSC constituent is denoted by s_2 (resp. s_3).

Definition 3: The multiplexer is an intelligent switch that distributes turbo coded bits s_i over the D_{st} parallel sub-channels of the BO-channel.

Actually, the multiplexer should be called "de-multiplexer" or equivalently "channel interleaver". We have chosen the word "multiplexer" in order to avoid any confusion with the interleaver denoted by π used inside the turbo code. Fig. 1 shows two important multiplexing examples. The two digits 1 and 2 represent the two states of the BO-channel. The symbol X represents a punctured parity bit.

Horizontal Multiplexer

$\mathbf{s_1}$	1	1	1	1	1	1
$\mathbf{s_2}$	2	X	2	X	2	X
$\mathbf{s_3}$	X	2	X	2	X	2

H-π-diagonal Multiplexer

$\mathbf{s_1}$	1	2	1	2	1	2
$\mathbf{s_2}$	2	X	2	X	2	X
$\pi^{-1}(\mathbf{s_3})$	X	1	X	1	X	1

Fig. 1. Multiplexers from [8]. Horizontal (top) and h-π-diagonal (bottom) multiplexers for a rate 1/2 parallel turbo code. Both multiplexers are suitable for a non-ergodic fading channel with $D_{st} = 2$ states.

Proposition 1: Let C be a rate $1/2$ parallel turbo code transmitted on a 2-state channel and built from $RSC(g_1(x), g_2(x))$. Under horizontal state multiplexing and for any input weight ω, the number η of codewords in C with incomplete state diversity is

$$\eta(\omega, d_{st} < 2) = 0 \quad \forall \ \ \omega \geq 2$$

Proof: For any non-zero turbo codeword, it is well-known that the Hamming weight of s_1 is $\omega \geq \omega_{min} = 2$ [3]. Also, the Hamming weight of both s_2 and s_3 must be positive despite puncturing. Hence, it is trivial that $d_{st} = 2$ since s_1 is always transmitted on the first channel state and (s_2, s_3) are transmitted on the second channel state. ∎

The recursive systematic convolutional constituent has constraint length $\nu + 1$. Its feedback generator polynomial is $g_1(x)$ and its forward generator polynomial is $g_2(x)$.

Definition 4: A recursive systematic convolutional code is said to be a *full-span* convolutional code if the generators satisfy $\deg(g_i(x)) = \nu$ and $g_i(0) = 1$, for $i = 1, 2$.
Trellis transitions outgoing from the 0-state and those incoming to the 0-state will be called *full-span transitions*,

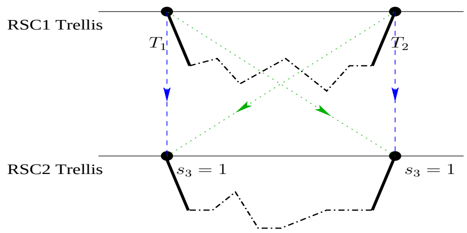

Fig. 2. Trellis error events for input weight $\omega = 2$. The two interleaving configurations are indicated. Diversity is guaranteed by full-span transitions.

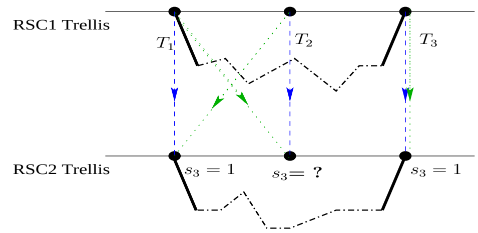

Fig. 3. Trellis error events for input weight $\omega = 3$. The six interleaving configurations are equivalent to two distinct configurations.

i.e. both bits are set to 1 on the transition label.

Proposition 2: Let C be a rate $1/2$ parallel turbo code transmitted on a 2-state channel and built from a full-span $RSC(g_1(x), g_2(x))$. Under h-π-diagonal state multiplexing and for any input weight ω, the number η of codewords in C with incomplete state diversity is

$$\eta(\omega, d_{st} < 2) = 0 \quad \forall \ \omega \geq 2$$

Proof: For $\omega = 2$ and $\omega = 3$: if a full-span transition is interleaved (via π) into a full-span transition, then state diversity is guaranteed. As shown in Figures (2) and (3), one of the full-span transitions in RSC1 is converted into a full-span transition in RSC2.
For $\omega \geq 4$: Consider the case where $\omega = 4$. Except for the unique interleaving configuration depicted in Fig. (4), all turbo codewords exhibit $d_{st} = 2$ due to full-span transitions. Now, let $\chi_i(s_j) \in \{1, 2\}$ denote the BO-channel state over which the binary element s_j belonging to RSC$_i$ is transmitted. We distinguish two cases when a critical configuration is transmitted on the channel.
Case 1: error event in RSC1 starts at state 1, $\chi_1(s_1) = 1$. Diversity is guaranteed by RSC1 because $\chi_1(s_2) = 2$.
Case 2: error event in RSC1 starts at state 2, $\chi_1(s_1) = 2$. Then, we distinguish two sub-cases:
Case 2.1: Information bit s_1 is set to 1 within the error event and hits state 1 yielding $\chi_1(s_1) = 1$. Hence, diversity is guaranteed by RSC1 without the help of RSC2.
Case 2.2: Information bit $s_1 = 1$ never hits state 1 in the trellis event of RSC1, $\chi_1(s_1) \neq 1$. This situation occurs because equality is not satisfied in (2) when $R_c = 1/2$ and $D_{st} = 3$, i.e. it is possible to create RSC1 codewords that never hit state 1. Thanks to the structure of the h-pi-diagonal multiplexer, at least one full-span transition in RSC2 has $\chi_2(s_3) = 1$ for $\chi_1(s_1) = 2$.
The same proof applies for $\omega > 4$. ■

Example with RSC$(7,5)_8$
Critical configurations: Let us give an example of critical configurations for $\omega = 4$ as defined in the proof of prop. 2. When $\chi_1(s_1) = 1$ and $\chi_1(s_2) = 2$, the RSC trellis is represented by the transition matrix

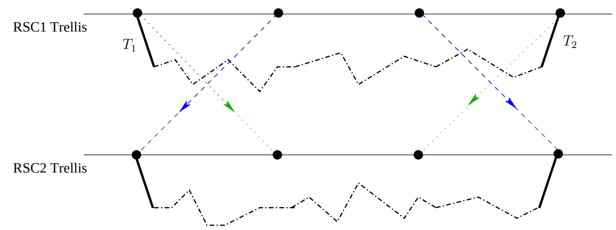

Fig. 4. A critical configuration for full-span outgoing and incoming transitions. Input weight $\omega = 4$.

$$A_1 = \begin{bmatrix} 0 & 0 & D_1D_2LW & 0 \\ D_1D_2LW & 0 & L & 0 \\ 0 & D_1LW & 0 & D_2L \\ 0 & D_2L & 0 & D_1LW \end{bmatrix}$$

When $\chi_1(s_1) = 2$ and $\chi_1(s_2) = X$, the transition matrix is

$$A_2 = \begin{bmatrix} 0 & 0 & D_2D_3LW & 0 \\ D_2D_3LW & 0 & L & 0 \\ 0 & D_2LW & 0 & D_3L \\ 0 & D_3L & 0 & D_2LW \end{bmatrix}$$

The complete weight enumerator $T(W, D, L)$ of simple error events is given by the top left entry of the product $A_1A_2A_1A_2\ldots$ or $A_2A_1A_2A_1\ldots$ depending on the position of the outgoing transition. A critical configuration is given by a product of type $A_2(A_1A_2)^\ell$ for an event of length $2\ell + 1$. For $\ell = 1\ldots3$ no critical configurations are found. For $\ell = 4$, we have

$$T(W, D, L) = \ldots + (2D_1D_2^5D_3^4 + D_2^8D_3^2)L^9W^4 + \ldots$$

Therefore, the shortest critical event for $\omega = 4$ has length $L = 9$. It includes 4 information bits with $\chi_1(s_1 = 1) = 2$, 4 parity bits with $\chi_1(s_2 = 1) = 2$, and 2 punctured bits with $\chi_1(s_2 = 1) = X$.
At this point, based on the study of η, the reader sees no difference between h-π-diagonal and horizontal multiplexers. Indeed, propositions (1) and (2) state that both multiplexers achieve full state diversity. The error rate performance depends on the achieved diversity and on the so-called *coding gain* or *product distance* defined by the product $\omega_1\omega_2$ of partial Hamming weights. Now, it should be clear that horizontal multiplexing shows a great unbalance between ω_1 and ω_2. As an example, for input weight $\omega = 2$, consider RSC(7,5)

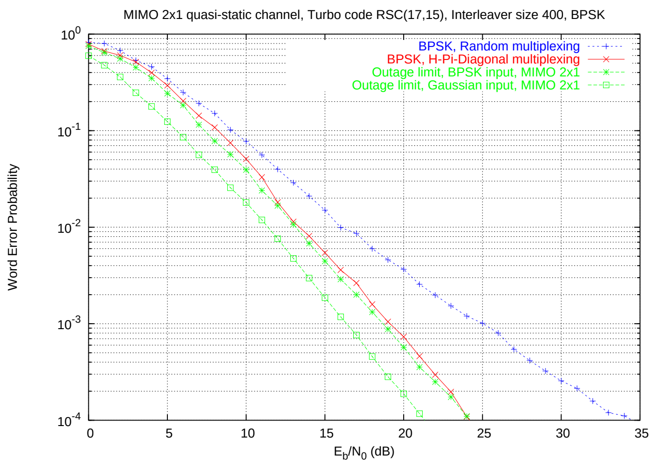

Fig. 5. BPSK modulation, $n_t = 2$, $n_r = 1$.

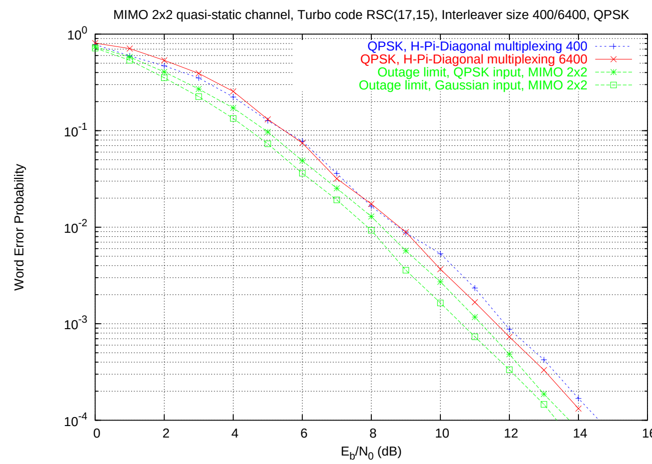

Fig. 7. QPSK modulation, $n_t = 2$, $n_r = 2$.

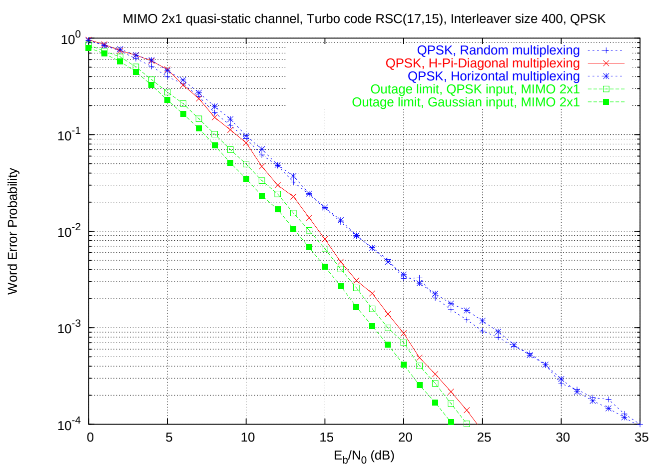

Fig. 6. QPSK modulation, $n_t = 2$, $n_r = 1$.

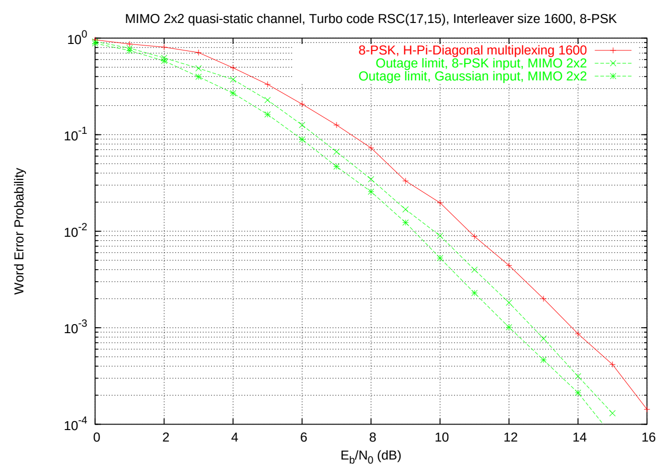

Fig. 8. 8-PSK modulation, $n_t = 2$, $n_r = 2$.

error events of length $L = 4 + 3i$ and total Hamming weight $w_H = 6+2i$, $i = 0 \ldots (N-4)/3$. For horizontal multiplexing, $\omega_1 = 2$ and $\omega_2 = 4 + 2i$. Therefore, its coding gain behaves as $O(N)$. For h-π-diagonal multiplexing, $\omega_1 = \omega_2 = 3 + i$. Hence, the coding gain of h-π-diagonal multiplexing increases as $O(N^2)$. The loss is even more dramatic for $\omega = 3$. The latter is neglected on the Gaussian channel since its contribution to the error rate performance is $O(1/N)$. On non-ergodic fading channels, when $\omega = 3$, turbo codewords satisfying $w_H(s_2) >> 1$ and $w_H(s_3) >> 1$ will suffer from the unbalance of horizontal multiplexing. A comparison between h-π-diagonal and horizontal multiplexers is illustrated in Fig. 6 with 2 transmit antennas and a QPSK modulation.

IV. Word Error Rate Performance (2Tx)

In this section, computer simulations are made for $n_t = 2$ and without linear precoding ($s = 1$) on the quasi-static MIMO channel. The rate 1/2 turbo code is built from RSC$(17, 15)_8$ and a pseudo-random interleaver π of size N. All curves include word error rate versus signal-to-noise ratio per bit. Fig. 5 shows the performance of a BPSK modulation with 2 transmit and 1 receive antenna, and $N = 400$. No further comment. Fig. 6 shows a similar situation with a QPSK modulation. The performance with 2 transmit and 2 receive antennas is given in Fig. 7. Notice that the word error rate is roughly the same for $N = 400$ and $N = 6400$. Finally, the performance of 8-PSK is illustrated in Fig. 8 and compared to both outage limits (discrete and Gaussian inputs).

V. Linear Precoding via DNA Rotations (4Tx)

In the case of $n_t = 4$ transmit antennas, we have $D_{st} = 4$. Maximum state diversity in (2) cannot be attained with $R_c = 1/2$ if $D_{st} = 4$. Therefore, we add a linear precoder in order to downgrade D_{st} from 4 to 2. This does not affect the physical channel diversity D_{ch}. If the rotation has $s = 4$, i.e. a full spreading unitary precoder as usually studied in the literature, then D_{st} will reduce to 1. Also, MIMO detection complexity increases exponentially with s. The solution to maintain $D_{st} = 2$ is given by Dispersive Nucleo Algebraic (DNA) precoders proposed in [12][13] for $s \leq n_t$. We build below a cyclotomic DNA rotation with a spreading factor equal to $s = 2$ time periods.

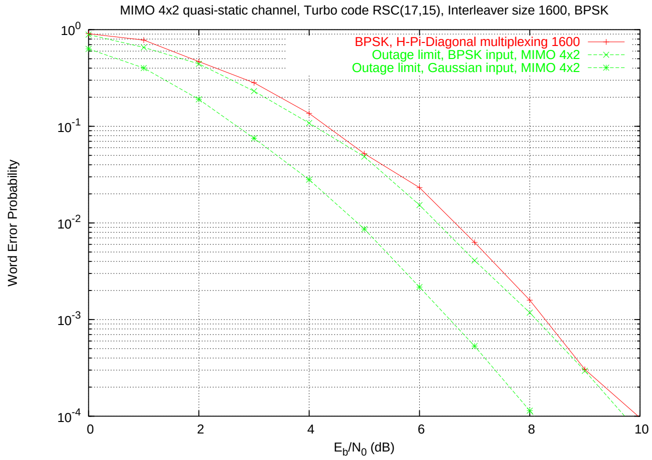

Fig. 9. BPSK modulation, $n_t = 4$ transmit antennas, $n_r = 2$ receive antennas. Linear precoding via a cyclotomic DNA rotation

The first step is to build a 4×4 precoder that is optimal under iterative decoding [11]. As an example, the cyclotomic unitary matrix S given in (V) satisfies the so-called *Genie conditions*:

- Vector $(\theta_{i,1}, \theta_{i,2})$ is orthogonal to vector $(\theta_{i,3}, \theta_{i,4})$ on any row i, $i = 1 \ldots 4$.
- Vectors $(\theta_{i,1}, \theta_{i,2})$ and $(\theta_{i,3}, \theta_{i,4})$ have equal norms.

The two Genie conditions can also be established by a maximum likelihood decoding analysis and the assumption of ideal channel interleaving [12][13].

$$S_{Cyclo} = [\theta_{ij}] = \frac{1}{2}\begin{bmatrix} 1 & 1 & e^{j6\pi/15} & -e^{j6\pi/15} \\ e^{j2\pi/15} & je^{j2\pi/15} & -e^{j8\pi/15} & je^{j8\pi/15} \\ e^{j4\pi/15} & -e^{j4\pi/15} & e^{j10\pi/15} & e^{j10\pi/15} \\ e^{j6\pi/15} & -je^{j6\pi/15} & -e^{j12\pi/15} & -je^{j12\pi/15} \end{bmatrix}$$

The second step is to place the orthogonal nucleotides inside an 8×8 matrix and separate them with null nucleotides. We obtain the following rotation for $n_t = 4$ and $s = 2$ (see proposition (2), page 54, in [12])

$$S_{DNA} = \begin{bmatrix} \theta_{11} & \theta_{12} & 0 & 0 & \theta_{13} & \theta_{14} & 0 & 0 \\ 0 & 0 & \theta_{11} & \theta_{12} & 0 & 0 & \theta_{13} & \theta_{14} \\ \theta_{21} & \theta_{22} & 0 & 0 & \theta_{23} & \theta_{24} & 0 & 0 \\ 0 & 0 & \theta_{21} & \theta_{22} & 0 & 0 & \theta_{23} & \theta_{24} \\ \theta_{31} & \theta_{32} & 0 & 0 & \theta_{33} & \theta_{34} & 0 & 0 \\ 0 & 0 & \theta_{31} & \theta_{32} & 0 & 0 & \theta_{33} & \theta_{34} \\ \theta_{41} & \theta_{42} & 0 & 0 & \theta_{43} & \theta_{44} & 0 & 0 \\ 0 & 0 & \theta_{41} & \theta_{42} & 0 & 0 & \theta_{43} & \theta_{44} \end{bmatrix}$$

Now, let us observe the MIMO channel with S_{DNA}. The QAM vector $z = (z_1, z_2, \ldots, z_8)$ goes through the precoder before H. Consider the lattice point zSH without adding Gaussian noise. The reader would notice that z_i is transmitted via the 1st and 2nd Tx antennas if i is odd, and via the 3rd and 4th Tx antennas if i is even. Consequently, the DNA precoder converts the $4 \times n_r$ MIMO channel onto two $2 \times n_r$ MIMO channels. Binary elements mapped to z_i when i is odd (resp. i is even) will be sent through the first BO-subchannel (resp. the second BO-subchannel). As a final illustration, Fig. 9 shows the error rate of BPSK modulation with 4 transmit and 2 receive antennas.

VI. Conclusion

We described simple space-time coding techniques for MIMO channels (2Tx and 4Tx) capable of achieving near outage limit performance. Low word error probabilities are obtained at signal-to-noise ratios extremely close to minimal achievable limits. Also, at fixed signal-to-noise ratio, the word error probability is insensitive to block length.

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