Physical-Layer Network-Coding over Block Fading Channels with Root-LDA Lattice Codes

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***Abstract*—We consider the problem of physical-layer network coding when the channel exhibits block fading. Specifically, we focus on the use of lattice codes in a compute-and-forward framework for realizing physical-layer network coding. We construct a novel lattice ensemble called the root-Low-Density Construction-A (root-LDA) ensemble which uses Construction A with root-low-density parity check (LDPC) codes. Using extensive simulations, we show that the proposed lattice codes exhibit full diversity when used over the block fading channels. In addition, their performance is comparable to the performance of LDA lattice codes optimized by the progressive edge growth algorithm over the additive white Gaussian noise AWGN channel. This suggests that root-LDA lattice codes provide a robust solution to the problem of implementing physical layer network coding over fading channels.**

I. Introduction

Physical-layer network-coding has drawn a lot of attention recently due to its ability to achieve significantly higher multiplexing gains as compared to conventional relaying strategies such as amplify-and-forward, compress-and-forward or decode-and-forward. One way to effectively combine coding and physical layer network coding is through the compute-and-forward framework which adopts lattice codes at the source nodes and directly computes an integer combination of codewords or linear combination of messages over finite field at each destination node [1], [2]. This paradigm has been studied intensively in the literature (see for example [3] [4] [5] and references therein).

Most of the work in the compute-and-forward literature focuses on the case where the channel stays fixed throughout the transmission, i.e., the channel is either an additive white Gaussian noise (AWGN) channel or the channel exhibits quasi-static fading. This assumption is too stringent to hold for some applications where the channel coherence time may be small and hence, a block fading model is more appropriate in such situations. Previous works on the use of lattice codes [1], [2] do not easily extend to the block fading case. On the one hand, modern wireless communication has taught us that coding schemes should exploit the diversity offered by block fading channels. On the other hand, block fading channels destroy the algebraic structure required to directly decode integer linear combinations of codewords. These conflicting phenomena make it difficult to predict whether physical layer network coding implemented through the compute-and-forward framework still provides substantial gains over other relaying strategies. In fact, naively using lattice codes which perform well under quasi-static fading often performs worse than amplify-and-forward in the presence of block fading. Indeed, one of the most significant open problems in this area is the design of compute-and-forward schemes that can effectively implement physical layer network coding (decode linear combinations) in the presence of block fading.

Very recently, Bakoury and Nazer have studied the block fading case [6] and derived (information-theoretically) achievable computation rates with infinite-dimensional lattice codes. In [6], the construction of practical computation codes to achieve full diversity is not investigated. Unlike [6], our focus here is on constructing practical coding schemes which exploit diversity for physical layer network coding (decoding linear combinations) under iterative decoding. To the best of our knowledge, this is the first result that shows a coding scheme that provides full diversity under iterative decoding for decoding linear combinations.

To this end, we propose a novel lattice ensemble which uses Construction A [7] with non-binary root-low-density parity check (LDPC) codes proposed by Boutros *et al.* in [8]. The motivation behind this choice is that binary root-LDPC codes have been shown to be able to achieve full diversity orders under iterative decoding. Thus, one expects lattices constructed upon non-binary root-LDPC codes can similarly provide high diversity orders. This new lattice ensemble is referred to as root-LDA lattice ensemble by following [9]. Extensive simulations are performed to verify the performance of the proposed root-LDA lattice codes. For point-to-point communication, we show that the proposed LDA lattice codes provide comparable performance as compared to conventional LDA lattice codes [9] optimized with progressive edge growth (PEG) algorithm [10] when used over an AWGN channel, whereas the proposed root-LDA lattice codes are able to provide full diversity under iterative decoding when used over block fading channel. We then perform simulations for decoding linear combinations and show that the proposed root-LDA lattice codes can indeed achieve full diversity under iterative decoding for both the two-way relay channel (TWRC)

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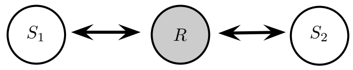

Fig. 1. The two-way relay channel.

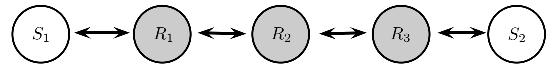

Fig. 2. The multiple-hop line network with 3 relay nodes

and the multiple-hop line network. Specifically, compute-and-forward with root-LDA lattice codes can surpass amplify-and-forward in the multiple-hop line network while compute-and-forward with lattice codes that are not optimized for block-fading channels cannot. Finally, it should be noted that a physical layer network coding scheme should be robust against different channel conditions, i.e., constant fading channel to fast fading channel (or somewhere in between). Simulation results suggest that the proposed codes exactly provide this kind of robustness.

The rest of the paper is organized as follows. In Section II, we state the problem of compute-and-forward over block fading channels. In Section III, the proposed root-LDA lattice ensemble is presented. We then apply the proposed root-LDA lattices for function computation over block fading channels in Section IV. This includes using the root-LDA lattices for both the TWRC and the multiple-hop line network. Section V concludes the paper.

A. Notations

Throughout the paper, $\mathbb{R}$ denotes the field of real numbers, $\mathbb{Z}$ denotes the set of integers, and $\mathbb{F}_p$ denotes finite field of order p. Vectors and matrices are written in boldface lowercase and boldface uppercase, for example $\mathbf{x}$ and $\mathbf{X}$, respectively.

II. Problem Statement

A. Two-way relay channel

In this paper, although our goal is to design computation codes with full diversity for compute-and-forward [1], we particularly focus on the first phase (uplink) of the TWRC for simplicity. It must be noted that the codes designed here are not limited to the TWRC and are applicable to compute-and-forward. As shown in Fig. 1, in this model, two source nodes S_1 and S_2 wish to exchange information via the relay node. Each source node S_i encodes its message $\mathbf{u}_i \in \mathbb{F}_p^K$ to the transmitted signal $\mathbf{x}_i \in \mathbb{R}^N$ that is subject to an input power constraint

$$\frac{1}{N}\|\mathbf{x}_i\|^2 = \frac{1}{N}\sum_{n=1}^{N}|x_i[n]|^2 \le P,\ \ i = 1, 2. \tag{1}$$

Unlike most of the work in the literature (see for example [2] [11]), we consider a scenario where the channel coherence time (l symbols) is less than the duration of N symbols. Source nodes and relay node are assumed to have perfect knowledge of channel coherence time. Hence, the channel can be modeled as a block fading channel with $b \triangleq N/l$ independent fades thus the signal received at relay is given by

$$\begin{aligned} y[n] &= h_{S_1}^{(j)} x_1[n] + h_{S_2}^{(j)} x_2[n] + z[n], \\ n &= 1, 2, \dots, N,\ \ j = 1 + \left\lfloor \frac{(n-1)}{l} \right\rfloor, \end{aligned} \tag{2}$$

where $h_{S_i}^{(j)} \in \mathbb{R}$ is fading coefficient of j-th sub-block between node S_i and R and $z[n] \sim \mathcal{N}(0, 1)$ is Gaussian noise. We write the collection of the jth block of channel coefficients as $\mathbf{h}^{(j)} = [h_{S_1}^{(j)}\ h_{S_2}^{(j)}]$, $j = 1, 2, \dots, b$. These channel coefficients are assumed to be perfectly known to the relay node but unknown to both the transmitters.

The relay then forms $\hat{\mathbf{w}}$ an estimate of the function $\mathbf{w} = b_1\mathbf{u}_1 \oplus b_2\mathbf{u}_2$ where $b_1, b_2 \in \mathbb{F}_p$ are chosen according to the channel vector $\mathbf{h}^{(j)},\ j = 1, 2, \dots, b$. The performance metric considered throughout the paper is the symbol error rate (SER) defined as

$$P_e \triangleq \mathbb{P}\left\{\hat{w}[n] \neq w[n]\right\}. \tag{3}$$

B. Multiple-hop line network

We consider information exchange on the line network shown in Fig. 2 where S_1 and S_2 wish to exchange information through three relays R_1, R_2, R_3 forming a line network. A two-way relay channel described in previous section consists a sequence of three nodes (N_1, N_2, N_3) where N_1 and N_3 are source nodes and N_2 is relay. The line network consists of three two-way relay channels which are (S_1, R_1, R_2), (R_1, R_2, R_3), and (R_2, R_3, S_2), i.e., codeword $\mathbf{x}_1$ sent from S_1 experiences 3 uplink channels and 1 downlink channel of TWRC to reach to S_2 and codeword $\mathbf{x}_2$ sent from S_2 experiences 3 uplink channels and 1 downlink channel of TWRC to reach to S_1.

III. Proposed Root-LDA Lattice Codes

In this section, we introduce the proposed root-LDA lattice ensemble and study the performance. We use both the point-to-point AWGN channel and the point-to-point block-fading channel to examine the performance of the proposed scheme. Note that the point-to-point block fading channel can be derived from (2) by fixing $\mathbf{x}_2[n] = 0$ for all n and the point-to-point AWGN channel can be obtained by further forcing $b = 1$. In what follows, we extend the root-LDPC ensemble proposed by Boutros *et al.* [8] to the non-binary case and then present the proposed root-LDA lattice ensemble.

A. Non-binary root-LDPC codes

We first review the family of binary root-LDPC codes which has been shown to be exhibit full diversity. We use its corresponding Tanner graph [12] to describe the parity check matrix of a root-LDPC code. The main idea behind root-LDPC codes is to design connecting edges such that each information bit would participate in rootchecks where all the other connected bits experience different fades and thus guarantee full diversity order. For the ease of exposition, we consider $b = 2$ and a regular $(3, 6)$ root-LDPC ensemble in the following. We will first demonstrate how to construct the

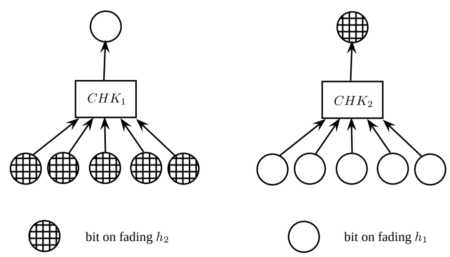

Fig. 3. Rootchecks of a root LDPC code.

Tanner graph of a binary root-LDPC code [8] and then extend it to the non-binary case.

In a binary root-LDPC code, full diversity is obtained by following the criteria of rootchecks:

1) There are 2 types of rootchecks denoted as CHK_1 and CHK_2, the number of rootchecks for each type is $N/4$.
2) Type 1(2) rootcheck is connected with 5 variable nodes experiencing fading h_2 (h_1) and 1 variable node experiencing fading h_1 (h_2).

The main idea of this design is shown in Fig. 3 where we can see that if only h_1 (h_2) is in deep fade, the information from the other 5 bit nodes experiencing h_2 (h_1) would make CHK_1 (CHK_2) reliable. So in order to make the code fails, both h_1 and h_2 have to be in deep fade. The parity-check matrix corresponding to Tanner graph thus designed is given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{H}_{2i} & \mathbf{H}_{2p} \\ \mathbf{H}_{1i} & \mathbf{H}_{1p} & \mathbf{I} & \mathbf{0} \end{bmatrix}, \tag{4}$$

where all the sub-matrices are $N/4 \times N/4$ matrices, $\mathbf{I}$ and $\mathbf{0}$ are the identity and the zero matrices, respectively, and the other four matrices are chosen to fulfill the degree profile. The columns of the sub-matrices $\mathbf{H}_{ji}$ and $\mathbf{H}_{jp}$ correspond to the information bits and parity bits experiencing h_j for $j \in \{1, 2\}$, respectively. We emphasize here that the first $N/4$ rows correspond precisely to the type 1 rootcheck and the second $N/4$ rows correspond to the type 2 rootcheck.

In order to construct lattices from root-LDPC codes, we first consider root-LDPC codes codes over $\mathbb{F}_p$. This can be easily done (but not necessarily optimally) by considering (4) as the skeleton of the parity check matrix of the non-binary code and randomly choosing a non-zero element in $\mathbb{F}_p$ for replacing each 1 in the skeleton matrix. We refer codes thus constructed as non-binary root-LDPC codes.

B. The proposed Root-LDA lattice code

We are now ready to state the proposed root-LDA lattices. This family of lattices is constructed by using Construction A [7] with non-binary root-LDPC codes. Specifically, let $\mathcal{C}$ be an N-dimensional non-binary root-LDPC code over $\mathbb{F}_p$ and use $\mathcal{M}$ the natural mapping to map the codeword symbols onto $\mathbb{Z}/p\mathbb{Z}$. Finally, tiling the results to the entire $\mathbb{R}^N$ to get a lattice of the form

$$\Lambda \triangleq \mathcal{M}(\mathcal{C}) + p\mathbb{Z}^N, \tag{5}$$

where the sum used above is the Minkowski sum. The fact that Λ is indeed a lattice can be easily shown by noticing that $\mathcal{M}$ is an isomorphism and $\mathcal{C}$ is a linear code. Also note that the name of the proposed lattice ensemble comes from [13] where di Pietro *et al.* use the term LDA lattices to refer lattices obtained from Construction A with regular LDPC codes.

So far, we have only discussed lattices instead of lattice codes. One can use the approach in [14] or [15] to build nested lattice codes. This approach first builds a pair of nested lattices (Λ_f, Λ_c) with $\Lambda_c \subset \Lambda_f$ and then use $\Lambda_f \cup \mathcal{V}(\Lambda_c)$ as codebook where $\mathcal{V}(\Lambda_c)$ denotes the fundamental Voronoi region of Λ_c.

In what follows, we discuss a decoding algorithm for the proposed root-LDA lattice. This includes a message-passing algorithm for determining the coset and a quantizer to quantize the received signal to the nearest lattice point inside the decoded coset. Let $\boldsymbol{\lambda} = \mathcal{M}(\mathbf{c}) + p\mathbf{k}$ be the lattice point transmitted. We first describe the message-passing algorithm whose goal is to determine $\mathbf{c}$ corresponding to $\boldsymbol{\lambda}$ as follows.

Initialization: Recall that the channel model is the point-to-point channel now. We first initialize the algorithm by computing the following a posteriori probability (APP) for each $n = 1, \dots, N$,

$$\mathbb{P}(c[n] = v|y[n]) = \sum_{\chi \in \Gamma_v} \mathbb{P}(h^{(j)}\chi|y[n]), \tag{6}$$

where $\Gamma_v = \{\chi|\chi = \mathcal{M}(v) + p\zeta, \zeta \in \mathbb{Z}\}$ and

$$\mathbb{P}(h^{(j)}\chi|y[n]) \propto \exp(-\frac{(y[n] - h^{(j)}\chi)^2}{2\sigma^2}), \tag{7}$$

where $j = 1 + \left\lfloor \frac{(n-1)}{l} \right\rfloor$. Note that in practice, one can choose the set Γ_v to include only a finite number of $\zeta \in \mathbb{Z}$ according to $y[n]$ for each n in order to get a good approximation. For example, one can choose Γ_v to include 3 such numbers that are closest to $y[n]$.

Message Update: After initialization is done, standard message update for variable nodes and check nodes over $\mathbb{F}_p$ is applied. The updated message for n-th variable node at m-th iteration denotes as

$$\mathbb{P}^{(m)}(c[n]|\mathcal{C}, \mathbf{y}\backslash y[n]). \tag{8}$$

Decision: At the end of this algorithm, the decision is made based on

$$\hat{c}[n] = \underset{v}{\operatorname{argmax}}\ \mathbb{P}^{(m)}(c[n] = v|\mathcal{C}, \mathbf{y}\backslash y[n]))\mathbb{P}(c[n] = v|y[n]). \tag{9}$$

Let $\hat{\mathbf{c}}$ be the output of the above message-passing algorithm. The decoder then looks into the coset $\mathcal{M}(\hat{\mathbf{c}}) + p\mathbb{Z}^N$ and quantize the received signal $\mathbf{y}$ to the nearest point inside this coset to form our estimate $\hat{\boldsymbol{\lambda}}$.

C. Simulation results

In this section, we perform simulations to demonstrate the performance of the proposed lattice codes and decoding algorithm. For the sake of simplicity, we use hypercube shaping to carve a lattice code from the proposed lattices. i.e., we set $\Lambda_c = p\mathbb{Z}^N$. The performance metric we consider is either codeword error rate ($\mathbb{P}(\hat{\mathbf{c}} \neq \mathbf{c})$) or SER ($\mathbb{P}(\hat{u}[n] \neq u[n])$) for $n \in \{1, \ldots, K\}$. We use SNR to denote average information symbol energy over N_0 in dB. The fine lattice Λ_f considered is a proposed root-LDA lattice with underlying code $\mathcal{C}$ being a $(3, 6)$ root-LDPC code over $\mathbb{F}_5$. For comparison, we also construct an LDA lattice code whose underlying code is a non-binary LDPC code over $\mathbb{F}_5$ with parity check matrix constructed by the PEG algorithm [10]. The codeword length is N=1200 and information length is K=600 for both codes. In Fig. 4, we consider the AWGN channel and plot the SER where we observe comparable performance between these two codes.

We then consider the block fading channel as shown in Fig. 5 and 6. In Fig. 5, we plot the codeword error rate and observe that the proposed root-LDA lattice code is roughly 8 dB better at rate 10^{-4} over the LDA lattice code, also the slope of root-LDA lattice code is 2 (as expected) while LDA lattice code does not show double diversity at that point. Similar observation can be made for the SER performance showing in Fig. 6 which 7 dB gain is observed at SER 10^{-4}, again the slope of the former is 2 while the latter does not show double diversity at that point. . This is mainly because the root-LDA lattice code designed specifically for fading channel can achieve full diversity while LDA lattice code is not optimized for fading channel. In Fig. 5, we also show the so called information outage probability [8] defined as

$$P_{out}(\alpha, R) = Pr\{\mathcal{I}(\alpha, \mathbf{h}) < R\}, \tag{10}$$

where α is signal-to-noise ratio and $\mathbf{h} = [h^{(1)} \ldots h^{(b)}]$, $\mathcal{I}(\alpha, \mathbf{h})$ is the instantaneous input-output mutual information between the input and output of the channel given

$$\mathcal{I}(\alpha, \mathbf{h}) = \frac{1}{b} \sum_{i=1}^{b} I_{AWGN,p}(\alpha {h^{(j)}}^2), \tag{11}$$

$I_{AWGN,p}(s)$ is the input-output mutual information of an AWGN channel with SNR s and input as $\mathbb{Z}$ hypercubically shaped by $p\mathbb{Z}^N$ [16]. This information outage probability is often used as a lower bound on codeword error rate. Comparing the performance of the proposed root-LDA lattice and the information outage probability, one can see that the proposed scheme has the right slope and hence exploit all the diversity offered by the channel. Also, the proposed scheme approaches (less than 2dB) the information outage probability.

IV. Root-LDA lattice Codes for Compute-and-Forward in Block Fading Channel

We now study using the proposed root-LDA lattice codes for the problem stated in Section II. Our main goal is to use such codes to achieve full diversity for function computation. Specifically, S_1 and S_2 adopt the same nested lattice code and map their messages $\mathbf{u}_1$ and $\mathbf{u_2}$ onto some lattice points $\mathcal{M}(\mathbf{c}_1) + p\mathbf{k}_1$ and $\mathcal{M}(\mathbf{c}_2) + p\mathbf{k}_2$, respectively. At the relay, the first thing must be done is to determine the function $\mathbf{w} = b_1\mathbf{u}_1 \oplus b_2\mathbf{u}_2$ according to the channel coefficients. To do this,

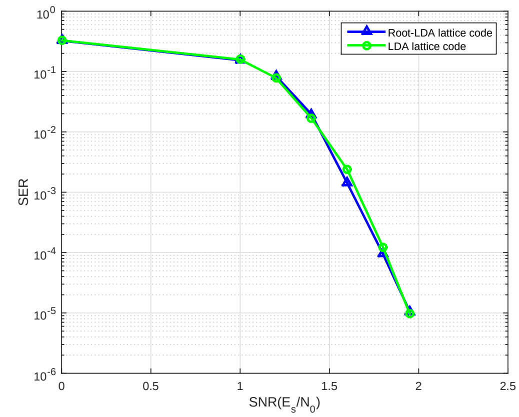

Fig. 4. SER performance for point-to-point AWGN channel.

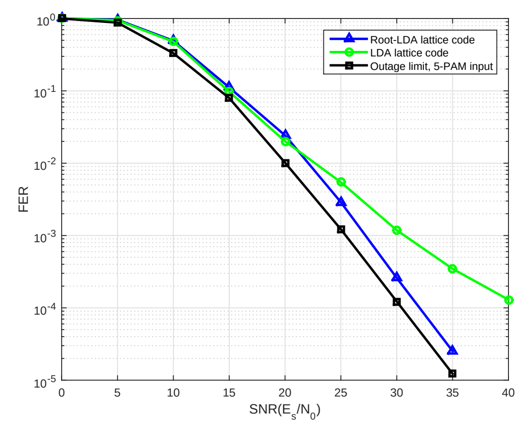

Fig. 5. Codeword error rate performance for point-to-point block fading channel with $b = 2$.

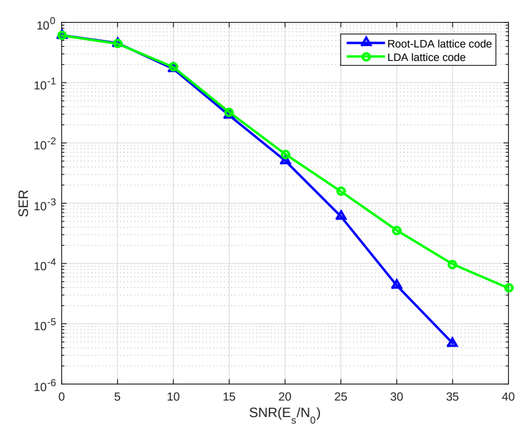

Fig. 6. SER performance for point-to-point block fading channel with $b = 2$.

we first find integer vector $\mathbf{a} = [a_1, a_2]$ such that the following average computation rate is maximized:

$$\mathcal{R}(\mathbf{h}^{(1)}, \ldots, \mathbf{h}^{(b)}, \mathbf{a}) = \frac{1}{b}\sum_{j=1}^{b} \frac{1}{2}\log^{+}\left(\left(\|\mathbf{a}\|^2 - \frac{P|\mathbf{h}^{(j)H}\mathbf{a}|^2}{1+P\|\mathbf{h}^{(j)}\|^2}\right)^{-1}\right), \qquad (12)$$

where $\log^{+}(x) \triangleq \max(\log(x), 0)$. Note that this rate expression can be easily derived by applying the (information-theoretic) result in [1] b times. Then, we set $b_1 = a_1 \mod p$ and $b_2 = a_2 \mod p$. Note that here we are not claiming that the above rate is achievable by the proposed scheme. The intuition of choosing the function this way is that typically, functions result in large rate in (12) would have low error rates.

Once the function $\mathbf{w}$ is determined, the relay first computes the APP for each $n \in \{1, \ldots, N\}$ given by

$$P_{APP}(v_1, v_2) = \sum_{\chi_1 \in \Gamma_{v_1}} \sum_{\chi_2 \in \Gamma_{v_2}} \mathbb{P}(h_{S_1}^{(j)}\chi_1 + h_{S_2}^{(j)}\chi_2 | y[n]), \quad (13)$$

where

$$\mathbb{P}(h_{S_1}^{(j)}\chi_1 + h_{S_2}^{(j)}\chi_2 | y[n]) \propto \exp\left(-\frac{(y[n] - h_{S_1}^{(j)}\chi_1 - h_{S_2}^{(j)}\chi_2)^2}{2\sigma^2}\right).$$

Given b_1, b_2, and APPs, we first try to decode the coset $\mathcal{M}(\mathbf{c}_f) + p\mathbb{Z}^N$ where $\mathbf{c}_f = b_1\mathbf{c}_1 \oplus b_2\mathbf{c}_2$ by computing for each $n \in \{1, \ldots, N\}$,

$$\mathbb{P}(c_f[n] = v | y[n]) = \sum_{b_1 v_1 \oplus b_2 v_2 = v} P_{APP}(v_1, v_2). \qquad (14)$$

The decoder then performs standard message update for check nodes and variable nodes over $\mathbb{F}_p$. The updated message for n-th variable node at m-th iteration is denoted as

$$\mathbb{P}^{(m)}(c_f[n] | \mathcal{C}, \mathbf{y} \backslash y[n]). \qquad (15)$$

The decision $\hat{c_f}[n]$ at m-th iteration is given by

$$\hat{c_f}[n] = \operatorname*{argmax}_{v} \mathbb{P}^{(m)}(c_f[n] = v | \mathcal{C}, \mathbf{y} \backslash y[n]) \mathbb{P}(c_f[n] = v | y[n]).$$

After determining the coset $\mathcal{M}(\hat{\mathbf{c}_f}) + p\mathbb{Z}^N$, we then quantize the received signal to the nearest lattice point inside this coset. Note that this last step is redundant if $p\mathbb{Z}^N \subseteq \Lambda_c$ as in this case, all the information we need is in $\mathbf{c}_f$.

A. Simulation results for TWRC

We now investigate the performance of using the proposed scheme for compute-and-forward (CF) through simulations. We first consider the TWRC and compare the SER at the relay of CF root-LDA lattice code, that of LDA lattice code, and amplify-and-forward (AF) with root-LDA lattice code. For all the schemes, we use hypercubic shaping and properly scale the signal to make the transmitted signal satisfy the power constraint. Note that although the main goal is only to study the compute-and-forward paradigm, we still consider the second (broadcast) phase of the TWRC here and in the following simulation for completeness. Assuming $\mathbf{c}_f$ is correctly computed, it is encoded to a codeword $\mathbf{x}_f$ subject to a power constraint P and forwarded back to the two nodes together with the knowledge of b_1 and b_2. The channel is assumed to be reciprocal. One can then follow standard approach in the literature (see [2] for example).

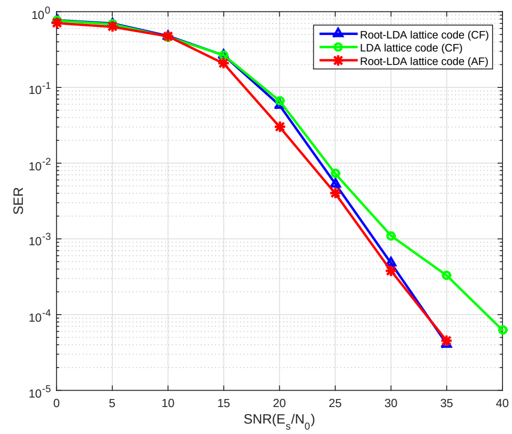

Fig. 7. End-to-end SER performance for TWRC with $b = 2$.

In Fig. 7, the simulation results for $b = 2$ is shown. One observes that both AF with root-LDA lattice code and CF with root-LDA lattice code provide diversity order of 2 while CF with LDA lattice code cannot achieve this diversity. Moreover, in this TWRC, AF outperforms CF as it does not decode the received signal at the relay and hence does not incur self-interference. However, the fact that AF does not compute (decode) also means that it cannot clean any noise. i.e., the noise gets accumulated. This drawback will become pronounced when we consider networks with multiple hops.

B. Simulation results for multiple-hop line network

We consider a protocol [2] shown in Fig. 8. $\mathbf{x}_{i,j}$ denotes j-th codeword sent from S_i. This protocol is in the initialization mode during the first 2 slots then turn to operating mode from the 3rd slots. The description of this protocol under the operating mode is as follows:

- In odd number slots, S_1, S_2, and R_2 transmit while rest of nodes listen
- In even number slots, S_1, S_2, and R_2 listen while rest of nodes transmit.
- S_1 and S_2 transmit new codewords whenever they allowed to transmit.

Note that the power constraint for source nodes S_i and relay nodes R_i is subject to P. In Fig. 8, we use an example to illustrate the transmission protocol. For the ease of presentation, we set the channel coefficients to be 1 for example; however, for the simulation shown later, we allow the channel to be block fading channel. In slot 1, the received signals at R_1 and R_3 correspond to $\mathbf{x}_{1,1}$ and $\mathbf{x}_{2,1}$ originated from S_1 and S_2, respectively, where we use $\mathbf{x}_{i,j}$ to denote the jth codeword from S_i. In slot 2, R_1 and R_3 forward $\mathbf{x}_{1,1}$ and $\mathbf{x}_{2,1}$, respectively, and R_3 computes the linear combination $\mathbf{x}_{1,1} + \mathbf{x}_{2,1}$. The above steps conclude the initialization process. We then consider the operating mode. We only look

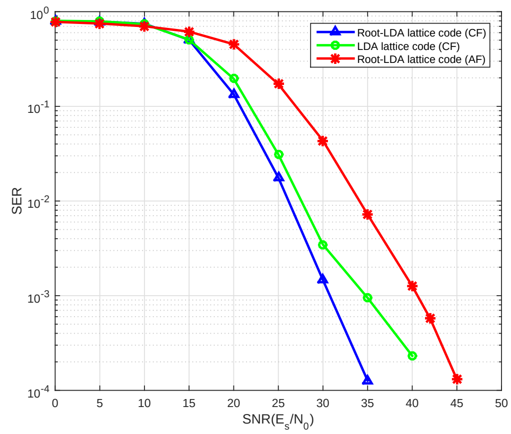

Fig. 9. SER performance of the 4-hop line network with $b = 2$.

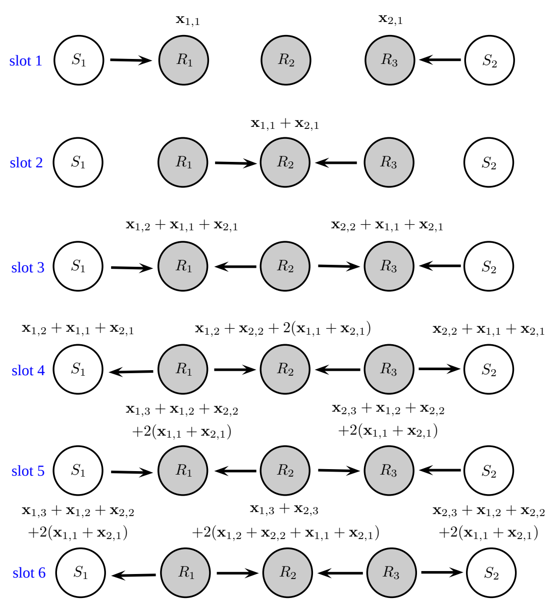

Fig. 8. The transmission protocol proposed in [2].

at information flow from S_1 to S_2 and the signal model for the other direction can be obtained similarly. In slot 3, R_2 forwards the previously computed codeword $\mathbf{x}_{1,1}+\mathbf{x}_{2,1}$ to R_3. Meanwhile, S_1 and S_2 transmit new codewords $\mathbf{x}_{1,2}$ and $\mathbf{x}_{2,2}$ to R_1 and R_3, respectively. R_3 then computes and the linear combination $\mathbf{x}_{2,2}+\mathbf{x}_{1,1}+\mathbf{x}_{2,1}$. In slot 4, R_3 forwards $\mathbf{x}_{2,2}+\mathbf{x}_{1,1}+\mathbf{x}_{2,1}$ to S_2 where $\mathbf{x}_{1,1}$ can be extracted out from this function and side information at S_2. In slot 5, R_2 forwards codeword $\mathbf{x}_{1,2}+\mathbf{x}_{2,2}+2(\mathbf{x}_{1,1}+\mathbf{x}_{2,1})$ to R_3. S_1 transmits codeword $\mathbf{x}_{1,3}$ to R_1. In slot 6, R_3 forwards $\mathbf{x}_{2,3}+\mathbf{x}_{1,2}+\mathbf{x}_{2,2}+2(\mathbf{x}_{1,1}+\mathbf{x}_{2,1})$ to S_2. S_2 can obtain $\mathbf{x}_{1,2}$ by cancel out side information and previously decoded codeword $\mathbf{x}_{1,1}$. This process keeps going until the end of transmission.

The same three schemes are adopted and their end-to-end SER performances are compared in Fig. 9. One can see that similar to the TWRC, the schemes with the proposed root-LDA lattice code enjoy diversity order of 2 while the one with LDA lattice code cannot. Moreover, the AF strategy perform worse than the CF ones due to the noise accumulation. The gap between CF and AF is expected to be larger as the number of hops increases.

V. Conclusion

We have proposed the family of root-LDA lattices which is obtained by Construction A with non-binary root-LDPC codes. We have then used these lattices to construct nested lattice codes for compute-and-forward over block fading channel. Extensive simulations have been performed to confirm that the proposed scheme achieves full diversity under iterative decoding for physical-layer network-coding and thus outperforms conventional lattice codes which do not take diversity order into account.

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